Multiply by Dividing

"What multiplies by dividing?" Mike challenged Jonathan with this question.

"That's impossible," replied Jonathan. "I know that multiplication and division are opposites. To multiply by dividing is nonsense. What's the answer?"

Mike laughed. "Bacteria, microbes, germs. You can't see them, but some of them can make you sick or even kill you. They are one-celled little bugs. Each bacterium divides into two cells every hour, or whatever."

"Wow, that's awesome! How many germs will a single cell split into in ten hours, if they divide every hour? It must be an awful lot."

Jonathan and Mike decided to figure out how many bacteria would result if a single cell divided in two every hour for ten hours. They made a table to keep track of their calculations.

When the boys had finished their table, they were really impressed. Over one thousand bacteria from just one cell in ten hours!

Try This

Complete the table for ten hours. How many bacteria are there at the end of ten hours?

Hours	Number of Bacteria
0	1
1	2
2	4
3	8
4	16

More Bugs

Mike and Jonathan wondered how many bacteria there would be at the end of twenty hours. Maybe they could find a shortcut. Doubling, or dividing into two parts, is the same as multiplying by two. How was the number of hours related to the number two?

They soon saw the pattern. For example:

At the end of two hours: 2 × 2 bacteria
At the end of three hours: 2 × 2 × 2 bacteria
At the end of six hours: the product of six 2s
At the end of ten hours: the product of ten 2s

They remembered that they knew an easy way to write these products. They could use *exponents*, with the number 2 as the base. They needed to figure out the powers of 2.

$$2 \times 2 = 2^2$$
$$2 \times 2 \times 2 = 2^3$$
$$2 \times 2 \times \ldots \times 2 \text{ (10 factors)} = 2^{10}$$
$$2 \times 2 \times \ldots \times 2 \text{ (20 factors)} = 2^{20}$$

To find the number of bacteria in twenty hours, they would need to find the product of twenty 2s, or 2^{20}.

But it was almost time for Mike to go home. "Let's just get an idea about how big that number is. We'll estimate it." They knew that 2^{10} was a little more than 1,000. They figured:

$$2^{20} \text{ is the same as } [2 \times 2 \times \ldots \text{ (10 factors)}] \times [2 \times 2 \times \ldots \text{ (10 factors)}],$$
$$\text{or } 2^{10} \times 2^{10}$$

They thought that a good estimate is 1,000 × 1,000 = 1,000,000. The actual number is a little more than a million bacteria at the end of twenty hours, and that's not even one day!

Try This

Find the exact number of bacteria at the end of twenty hours.

Bugs Multiply Even Faster

When Mike saw a headline in the newspaper about bacteria in hospitals, he read the article and told Jonathan about it. "I read about bacteria that doubled every half hour, not every hour. The article called a half hour the doubling time."

Jonathan was surprised. "That's even worse than we thought. How many bacteria can one of these bacteria cells produce in ten hours? Do you think it would be twice as many as the bacteria we talked about yesterday, the kind that doubled every hour?"

The boys decided to find out. Again they made a table. They called the new bacteria Type B and yesterday's bacteria Type A. This time they wrote the base two with the correct exponent so that they could discover the pattern.

Hours	Number of Type B Bacteria
0	1
½	2 = 2
1	4 = 2^2
1½	8 = 2^3
2	16 = 2^4

They could see that in two hours one Type B cell could produce as many bacteria as Type A had in four hours. It took only half the time for Type B to arrive at the same number of bacteria as Type A.

Try This

1. Continue the Type B table for five hours. How many bacteria have been produced at the end of five hours?
2. Estimate the number of Type B bacteria at the end of ten hours.

Think About This

The number of *E. coli* bacteria can double every twenty minutes. How long will it take for one *E. coli* cell to produce about a thousand bacteria? Estimate the number of bacteria at the end of ten hours.